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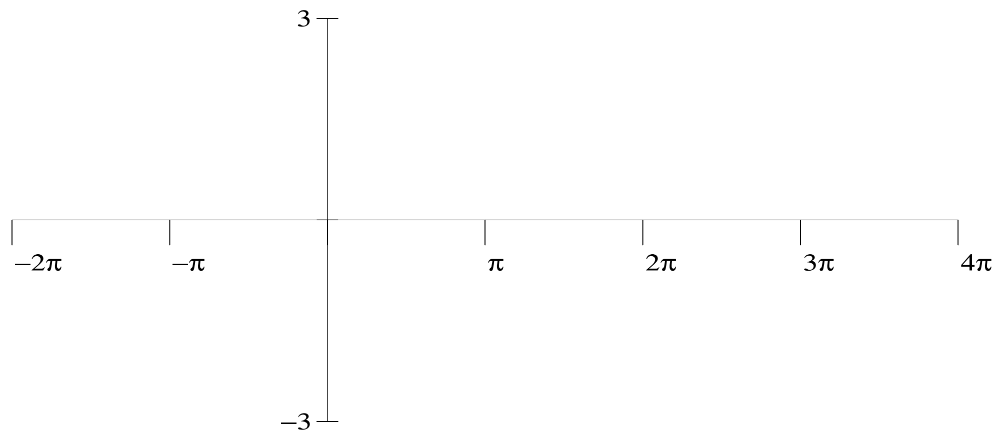
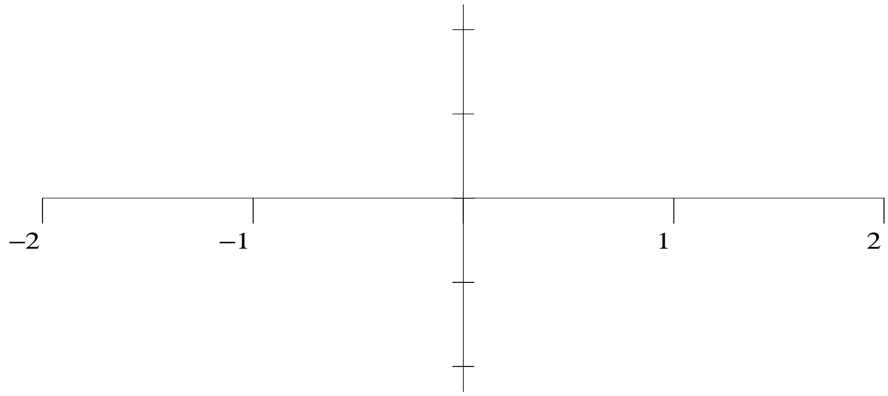
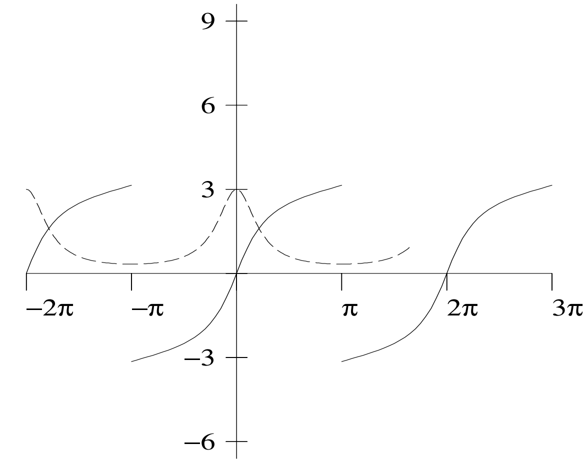


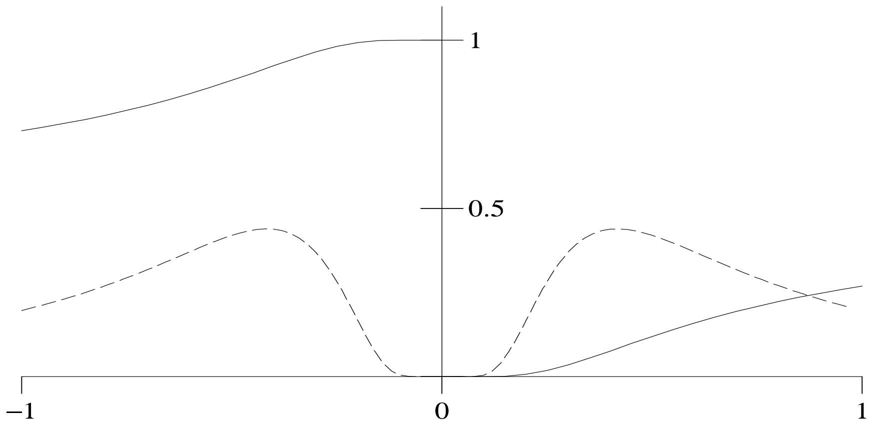
3
−3
−2π
−π
π
2π
3π
4π



−2
−1
1
2



9
6
3
−3
−6
−2π
−π
π
2π
3π



Figur 4. Th functions in quation (10). – – – , th int grand. ——, th discontinuous int gral.

and substituting this into (7) and combining inv rs tang nts giv s th following compact variation on (7):

$$\int \frac{3\,dx}{5 - 4\cos x} = x + 2\arctan\frac{\sin x}{2 - \cos x}\ . \qquad (\ )$$

A g n ralization of this xpr ssion that is suitabl for int gral tabl s is as follows. If

$$p^2 > q^2 + r^2\ ,$$

th n

$$\int \frac{dx}{p + q\cos x + r\sin x} = \frac{x}{\Delta} + \frac{2}{\Delta}\arctan \qquad r\cos x - q\sin x$$

int grabl , and not n c ssarily continuous, and h nc cov rs pi c wis continuous functions. Th c ntral topic of this pap r, nam ly th importanc of nsuring that int grals ar continuous, is an important factor in th d v lopm nt of corr ct rul s for int grating pi c wis -d fin d functions.

Th H avisid st p function is d fin d by

$$H(x) = \begin{cases} 1, & \text{for } x \geq 0 \; ; \\ 0, & \text{for } x < 0 \; . \end{cases}$$

As an xampl of how w want to work with this function, consid r th following diff r ntial quation coming from l m ntary b am th ory. Th b nding mom nt [illegible](x) in a b am xt nding from $x = 0$ to $x = l$ and supporting point loads P_a and P_b at $x = a$ and $x = b$ is giv n by th quation

$$\frac{d[illegible]}{dx}$$

and th boundary conditions giv th sam solution as b for . Th d fl ction can now b asily obtain d by int grating twic mor to find

$$y = \frac{1}{6}(x-a)^3 P_a H(x-a) + \frac{1}{6}(x-b)^3 P_b H(x-b) + \frac{1}{6}Kx^3 + Bx + C\ .$$

Th boundary conditions $y(0) = y(l) = 0$ giv us B and C. This m thod of solution is mor conv ni nt than th first on for p opl working by hand, and vastly mor conv ni nt for thos using an alg bra syst m.

5. Conclusions. In calculus t xtbooks, it is popular to includ a s ction on th us of int gral tabl s. In vi w of th r sults in s ction 2, th t xtbooks should warn stud nts that any xpr ssion xtract d from a tabl might contain a spurious discontinuity. With qual forc , w should r quir th   ditors of handbooks to ch ck th ir tabl s thoroughly. In ff ct, an ntry in a tabl should not b consid r d corr ct unl ss it is continuous on as wid an int rval as possibl . Th alt rnativ would b to not th int rval upon which th int gral is valid, without att mpting to broad n it, but this would b l ss us ful to th r ad r. Similar comm nts can b appli d to comput r alg bra syst ms.

Th W i rstrass substitution discuss d in s ction 3 was onc a standard topic in calculus t xtbooks, alb it an advanc d topic. It app ars l ss fr qu ntly now, but if it is tr at d, I think that an analysis of th discontinuity and its corr ct handling must b includ d. Th mat rial of s ction 4 com s from my xp ri nc of watching stud nts tackl probl ms such as th on d scrib d, and from impl m nting th solution on alg bra syst ms.

This pap r springs dir ctly from discussions with David Stout my r and Al Rich, th d v l- op rs of *Derive*, a comput r alg bra program. Almost all of th it ms discuss d h r hav b n impl m nt d in *Derive*, and I am grat ful to its d v lop rs for th ir int r st. I am also grat ful to th d v lop rs of *Maple* for stimulating discussions, and for adopting som of th id as abov .

References.

1. J. St wart, *Calculus, 2nd Ed.* Brooks/Col , Pacific Grov , CA, 1991.
2. W.H. B y r, *Handbook of Mathematical Sciences, 6th Ed.* CRC Pr ss, Boca Raton, FL, 19 7.
3. I.S. Gradsht yn and I.M. Ryzhik, *Table of Integrals, Series and Products,* Acad mic Pr ss, San Di go, CA, 1979.
4. M. Abramowitz and I.A. St gun, *Handbook of Mathematical Functions,* Dov r, N w York, NY, 1965.
5. D.J. J ffr y and A.D. Rich, Th   valuation of trigonom tric int grals avoiding spurious discon- tinuiti s, *ACM Trans. Math. Software*, submitt d.
6. M.W. Botsko, A fundam ntal th or m of calculus that appli s to all Ri mann int grabl func- tions, *this* MAGAZINE , **64** (1991), 347–34 .