An Algebraic Method for Analyzing Open-Loop Dynamic Systems

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Abstract. Maple Dynaflex RifSimp Dynaflex RifSimp

such as a four-bar mechanism, in which the components connect back to the structure being considered.

When a structure forms a closed loop, then Dynaflex will generate constraint equations that describe the drop in the degrees of freedom that accompanies the closing of a loop. From the point of view of this paper, we have discovered that the RifSimp package takes a great deal of time and memory to analyze closed-loop systems, but can make good progress with open-loop ones. This is what is reported here.

2 Example System Analysed Using Dynaflex

In order to keep the examples within printable limits, we shall use a simple spinning top as an example. A top is an axisymmetric body that spins about its body-fixed symmetry axis. It can precess about a vertical (Z) axis, and nutate about the rotated X axis. Figure 1 shows gravity acting in the $-Z$ direction. The center of mass is located at C, and the spinning top is assumed to rotate without slipping on the ground; this connection is modelled by a spherical (ball-and-socket) joint at O. The joint coordinates at O are represented by Euler angles (ζ, η, ξ), in the form of 3-1-3 Euler angles, meaning that they correspond to precession, nutation, and spin, respectively.

2.1 T S . G a

The system graph for the top is shown in Figure 2. The graph consists of nodes and edges. The nodes correspond to centres of coordinates, while the edges describe the system. Thus in the figure, we see nodes6.7(w)f75(ho2(e Tc [(002 T1(1,2,3e)-47861(whac)27.

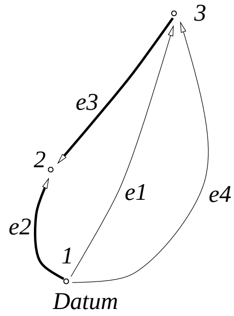

Fig. 2. [illegible]

that the body is a top is described by edge e1 being specified as a rigid body with a diagonal moment of inertia. The position of the centre of mass is specified by e3, which formally is a rigid-arm element. Finally, gravity is defined by e4, being a force element.

It is important to note that the graph consisting of {e2, e3} is denoted by heavy lines, whereas elements e1 and e4 are drawn in light lines. The elements {e2,e3} correspond to the open loop referred to above.

2.2 Ma[illegible] I[illegible] F[illegible]

The Maple input file corresponding to the above system is as follows.

[illegible]

2.3 T[illegible] S[illegible] b[illegible]c E[illegible]

The equations produced by Dynaflex from the above input file are presented below. By default, Dynaflex assigns its own notation for quantities such as Euler angles, moments of inertia, etc. Although Dynaflex notation is convenient of its internal programming, it results in equations which are di cult, and even ugly, to read when printed out for human use. Therefore we have edited the raw Dynaflex output format to simplify the notation to bring it in line with what human readers are used to seeing. The centre of mass is a distance l from the point of contact, the mass is m, the moment of inertia about the symmetry axis is C, and about a perpendicular axis is A. The Euler angles are given above.

$$C\ddot{\zeta}(t)\cos^2\eta(t) + C\ddot{\xi}(t)\cos\eta(t) - A\ddot{\zeta}(t)\cos^2\eta(t) - C\dot{\eta}(t)\dot{\xi}(t)\sin\eta(t)$$
$$-2C\dot{\eta}(t)\dot{\zeta}(t)\sin\eta(t)\cos\eta(t) + 2A\dot{\eta}(t)\dot{\zeta}(t)\sin\eta(t)\cos\eta(t) + A\ddot{\zeta}(t) = 0\,, \quad (1)$$
$$-mg\sin\eta(t) - A\dot{\zeta}^2(t)\sin\eta(t)\cos\eta(t) + A\ddot{\eta}(t) + C\dot{\xi}(t)\dot{\zeta}(t)\sin\eta(t)$$
$$+C\dot{\zeta}^2(t)\sin\eta(t)\cos\eta(t) = 0\,, \quad (2)$$
$$-C(-\ddot{\xi}(t) - \ddot{\zeta}(t)\cos\eta(t) + \dot{\eta}(t)\dot{\zeta}(t)\sin\eta(t)) = 0\,. \quad (3)$$

3 Automatic Symbolic Simplification Using RifSimp

The package RifSimp can process systems of polynomially nonlinear PDEs with dependent variables $u_1, u_2, \ldots u_n$, which can be functions of several independent variables. For the present application the only independent variable is time. The variables u_i can obey differential equations of varying order. RifSimp takes as its input a system of differential equations and a ranking of dependent variables and derivatives. RifSimp orders the dependent variables lexicographically[1] and the derivatives primarily by total derivative order:

$$u_1 \prec u_2 \prec \ldots \prec u_n \prec u_1' \prec u_2' \prec \ldots \prec u_n' \prec u_1'' \prec \ldots \quad (4)$$

Then equations are classified as being either leading linear (i.e. linear in their highest derivative with respect to the ordering $\prec$) or leading nonlinear (i.e. nonlinear in their highest derivative).

RifSimp proceeds by solving the leading linear equations for their highest derivatives until it can no longer find any such equations. Leading nonlinear equations (the so-called constraints), are treated by methods involving a combination of Gröbner Bases and Triangular Decomposition. It differentiates the leading nonlinear equations and then reduce them with respect to the leading linear equations. If zero is obtained, it means the equation is included in the ideal generated by the leading linear equations. If not, it means that this equation is a new constraint to the system. This is repeated until no new constraints are found.

T : (Output RifSimp form)
If v is a list of derivatives and w is a list of all derivatives (including dependent variables) lower in ranking than v, then the output of RifSimp has the structure

$$v = f(t, w) \quad (5)$$

subject to a list of constraint equations and inequations

$$g(t, w) = 0, \quad h(t, w) \neq 0 \quad (6)$$

P . See [6] and references therein.

[1] [illegible]

T. : (Existence and Uniqueness)
Given an initial condition $g(t^0, w^0) = 0, h(t^0, w^0) \neq 0$, there is a local analytic solution with this initial condition.

P . See [6].

4 Application of RifSimp to Example Problem

In order to apply RifSimp to the example above, we proceed as follows.

1. Change coordinates using $\cos\eta = \frac{1- \ (t)^2}{1+ \ (t)^2}, \sin\eta = \frac{2 \ (t)}{1+ \ (t)^2}$ to get a rational polynomial di erential system instead of trigonometric nonlinear di erential system. (RifSimp does not allow trigonometric functions.)
2. Give this polynomial di erential equation system to RifSimp, specifying the case split option to get normalized di erential equations.
3. Use RifSimp case tree to analyze the di erent cases.

With no assumptions on t1.

5 Some Special Cases

The importance of the