Getting from x to y without crashing: Computer syntax in mathematics education

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When we use technology to teach mathematics, we hope to focus on the mathematics, restricting the computer software systems to providing support for our pedagogy. It is a matter of common experience, however, that students can become distracted or frustrated by the quirks of the particular software system being used. Here, experience using the systems can, if they do not want to crash. The same is true of mathematical software.

A solution: It is better therefore to start the very first laboratory session with the instructions:

1. Chose the directory (folder) where you will save your work.
2. Make that directory (folder) the current directory. This can be done easily using the current directory bar in the Matlab window.
3. Open a new file.
4. Type a comment line with the ry (fo

1. The function is called with an array of values. It is important to know that Matlab functions such as can call a function with an array of values.
2. 1+x and 2+x evaluate to arrays.
3. x/(1+x) evaluates to a 1x1 matrix. 1./(2+x) evaluates to an array.
4. The 1x1 matrix is re-interpreted as a scalar.
5. The scalar and the array are combined together by using the above rule for adding a scalar and an array.
6. A plausible, but wrong, array is returned by the function.
7. The student cannot see the error or understand it and is frustrated because the answer to the problem is wrong.

A solution: Asking questions along the lines "How will Matlab evaluate and and " can encourage students to be more sensitive to the differences between the two constructions. My preference is to include such questions on assignments and then repeat them on exams; other teachers may prefer other approaches.

Problem: sometimes a solution, sometimes not. Matlab tries to be helpful to professionals by automatically returning approximate solutions to inconsistent equations. This can confuse students who are struggling to digest the differences between systems with one solution, families of solutions or no solutions.
Let us take some specific examples. Consider a set of equations having no solution, i.e., that are inconsistent:

$$x+y = 1,$$
$$x+y = 2.$$

We can convert this into the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Then we can try to solve this using the Matlab command

```
[1 1; 1 1] \ [1; 2]
```

This will produce an error message from Matlab that the matrix is singular. However, now we try a slightly more difficult problem:

$$x+y = 1,$$
$$x+y = 2,$$
$$x+2y = 3.$$

This becomes the matrix equation

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

It is probably still the case that in a first course on linear algebra, the instructor would like the students to recognize that this system is inconsistent. However, if students try using Matlab to verify their ideas, they might type

```
[1 1; 1 1; 1 2] \ [1; 2; 3]
```

and obtain the "solution" , which happens to be an approximation to a solution based on the theory of least squares.

A solution: There are two ways to handle this problem. The first way is to include the theory of least squares in the curriculum of the course. Many instructors object to this, however, because it is an admission that software, rather than the instructor's view of mathematics, is dictating the contents of the course. The second way is to warn students explicitly that they must not use the Matlab operator \ to decide consistency and inconsistency. In fact, it might be best to require every solution to be obtained using the RREF (Reduced Row Echelon Form) command explicitly. This means that Matlab is emulating the methods used by a human without Matlab.

Problem: function names. A first course on Linear Algebra that is using Matlab will probably not require the use of functions. Since the object of using Matlab is to illustrate mathematics, rather than teach programming, this is understandable. At second year, however, when we teach differential equations, functions become important.
The Matlab syntax for a function is the keyword followed by the output variables of the function, an "equals" sign, a name and then a list of input variables. Thus

```
function [x,y]=Name(a,b)
x=a+b;
y=a-b;
```

This function is written in the editor window and then saved to disk as an ".m" file. In this case "Name.m" would be pre-selected by the editor. A difficulty for a student arises if they now use this function as a model for others. It now is possible for a student to write a function

development. The first way is as an expression. It is the earlier method (meaning it was used in the first versions of Maple) and it is simpler to start with. Thus we write

To use this expression, we now use the commands

. An important point is that if we write