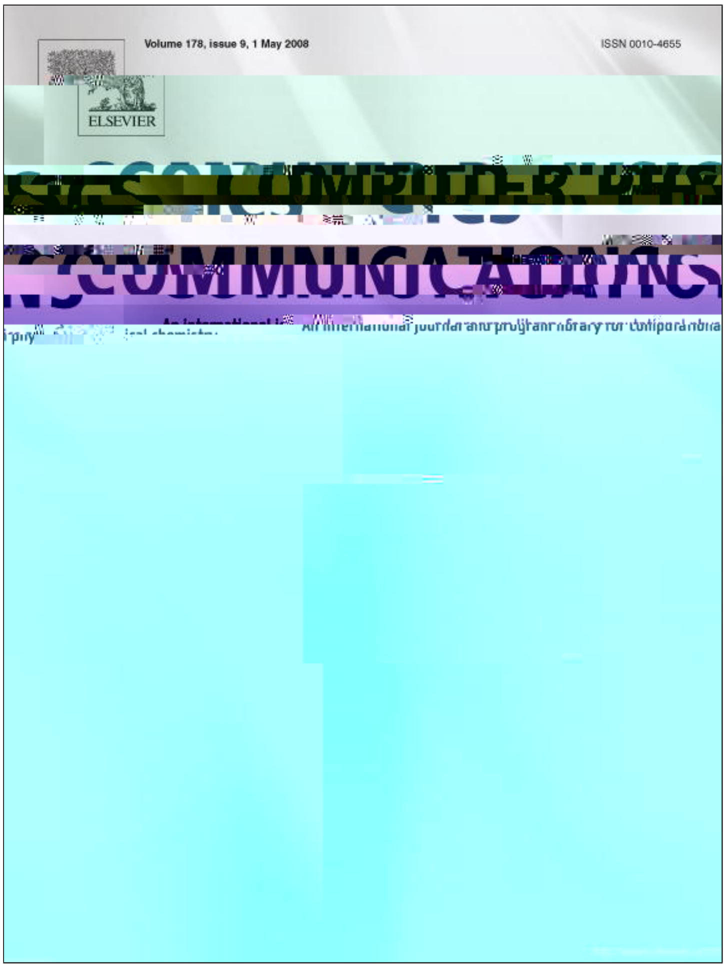
Volume 178, issue 9, 1 May 2008
ISSN 0010-4655
ELSEVIER
COMPUTER PHYSICS
COMMUNICATIONS

[5] W. Malfliet, W. Hereman, Phys. Scripta 54 (1996) 563.
[6] E.J. Parkes, B.R. Duffy, Comput. Phys. Comm. 98 (1996) 288.

02.30.Jr; 02.70.Wz; 01.50.Ff

Nonlinear partial differential equation; Travelling wave solution; Tanh-function method; Symbolic computation; Exact solution

1. Introduction

Mathematical modelling of physical systems often leads to nonlinear partial differential equations (PDEs). Explicit solutions, especially travelling wave solutions, to such equations are of fundamental importance. In order to help physicists and engineers better understand the mechanisms that govern these physical phenomena, many powerful and direct methods have been proposed. Among these are direct integration (whenever possible), Hirota's bilinear method [4], the inverse scattering transform [4], Painlevé expansion method [2] and the real exponential method [12,13].

A less sophisticated but more direct method, the tanh-function method, was proposed to find explicit travelling wave solutions to nonlinear PDEs. This method was due to Malfliet and Hereman [17,18]. Since then, a lot of relative contributions have been reported in dT1_1 1iattatu60 Tc 21iattatu6.de imec8(the)-4l.48(Comm.1 Tc 27.559 0 Td([17,18])Tj2ae33Sl.4y5en1.232 Td[ion)]TJ/T1_

```
> read "C:/pde/TWS.mpl"
```

The calling sequence is: TWS([illegible], function = [illegible], parameter = [illegible], [illegible]), where

- [illegible]—a PDE as described in (1);
- [illegible]—(optional) a list of functions to be used for finding the travelling wave solutions to [illegible], and the default value is: [rational, exp, sinh, csch, cosh, sech, tanh, coth, sin, csc, cos, sec, tan, cot, JacobiCN, JacobiSN];
- [illegible]—(optional) a list of parameters occurred in [illegible], and the default value is the empty list;
- [illegible]—(optional) a boolean value which determines the output of the solutions: if it is [illegible] (the default value), only real solutions are returned, otherwise, all solutions are returned.

Before comparing the new package with other existing packages, we mention a key point when using it. If the given PDE [illegible] contains parameters, and [illegible] has travelling wave solutions only for special parameter values, then these parameters must be specified by the argument parameter = [illegible] in the calling sequence above. Otherwise, the new package will treat these parameters as

- 1 sin/csc type solution:

$$(\ ,\ ) = \frac{\sqrt[3]{14}}{2}\left(\left(\sin\left(\frac{3}{4}\sqrt{2}\ \ +\ \ \right)\right)^{-1}\right)^{2/3}.$$

- 12 tanh/coth type solutions:

$$(\ ,\ ) = \frac{\sqrt[3]{4}}{4}\left(-\frac{(-1+\tanh(\frac{3}{56}\sqrt{14}\ \ -\frac{33}{56}\ \ +\ \ ))^2}{\tanh(\frac{3}{56}\sqrt{14}\ \ -\frac{33}{56}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{\sqrt[3]{2}}{2}\left(\frac{-1+\tanh(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ )}{\tanh(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(4+4\tanh\left(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ \right)\right)^{2/3},$$

$$(\ ,\ ) = \frac{\sqrt[3]{2}}{2}\left(\frac{1+\tanh(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ )}{\tanh(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(\frac{4-4\tanh(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ )}{\tanh(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(-4-4\tanh\left(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ \right)\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(4-4\tanh\left(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ \right)\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(\frac{-4-4\tanh(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ )}{\tanh(\frac{3}{28}\sqrt{14}\ \ +\frac{33}{28}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{1}{4}\left(-4+4\tanh\left(\frac{3}{28}\sqrt{14}\ \ -\frac{33}{28}\ \ +\ \ \right)\right)^{2/3},$$

$$(\ ,\ ) = \frac{\sqrt[3]{4}}{4}\left(\frac{(-1+\tanh(\frac{3}{56}\sqrt{14}\ \ -\frac{33}{56}\ \ +\ \ ))^2}{\tanh(\frac{3}{56}\sqrt{14}\ \ -\frac{33}{56}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{\sqrt[3]{4}}{4}\left(-\frac{(1+\tanh(\frac{3}{56}\sqrt{14}\ \ +\frac{33}{56}\ \ +\ \ ))^2}{\tanh(\frac{3}{56}\sqrt{14}\ \ +\frac{33}{56}\ \ +\ \ )}\right)^{2/3},$$

$$(\ ,\ ) = \frac{\sqrt[3]{4}}{}$$

```
> pde2:=diff(u(x,t),t)-(diff(u(x,t)^2,x$2)-p*u(x,t)+q*u(x,t)^2=0;
```

$$2 := \frac{\partial}{\partial}\ (\ ,\ ) - 2\left(\frac{\partial}{\partial}\ (\ ,\ )\right)^2 - 2\left(\frac{\partial^2}{\partial\ ^2}\ (\ ,\ )\right)\ (\ ,\ ) -\ \ (\ ,\ ) +\ \left(\ (\ ,\ )\right)^2 = 0$$

```
> TWS(pde2,function=[tanh]);
```

$$(\ ,\ ) = \frac{2\ \tanh(\frac{1}{4}\sqrt{\ }\ - \frac{1}{4}\ \ + \ )}{(-1 + \tanh(\frac{1}{4}\sqrt{\ }\ - \frac{1}{4}\ \ + \ ))},$$
$$(\ ,\ ) = \frac{-4\ \tanh(\frac{1}{8}\sqrt{\ }\ - \frac{1}{8}\ \ + \ )}{(1 - 2\tanh(\frac{1}{8}\sqrt{\ }\ - \frac{1}{8}\ \ + \ ) + (\tanh(\frac{1}{8}\sqrt{\ }\ - \frac{1}{8}\ \ + \ ))^2)},$$
$$(\ ,\ ) = \frac{2\ \tanh(\frac{1}{4}\sqrt{\ }\ + \frac{1}{4}\ \ + \ )}{(1 + \tanh(\frac{1}{4}\sqrt{\ }\ + \frac{1}{4}\ \ + \ ))},$$
$$(\ ,\ ) = \frac{4\ \tanh(\frac{1}{8}\sqrt{\ }\ + \frac{1}{8}\ \ + \ )}{(1 + 2\tanh(\frac{1}{8}\sqrt{\ }\ + \frac{1}{8}\ \ + \ ) + (\tanh(\frac{1}{8}\sqrt{\ }\ + \frac{1}{8}\ \ + \ ))^2)},$$
$$(\ ,\ ) = \frac{2}{(1 + \tanh(\frac{1}{4}\sqrt{\ }\ + \frac{1}{4}\ \ + \ ))},$$
$$(\ ,\ ) = \frac{-2}{(-1 + \tanh(\frac{1}{4}\sqrt{\ }\ - \frac{1}{4}\ \ + \ ))}.$$

Again, the package RAEEM obtains no solutions, while the package TWSolutions returns three nontrivial tanh type solutions.

```
> TWSolutions(pde2,function=[tanh],remove_redundant=true);
```

$$(\ ,\ ) = -,$$
$$(\ ,\ ) = \frac{-2}{(-1 + \tanh(-\_C1 + \frac{1}{4}\sqrt{\ }\ - \frac{1}{4}\ \ ))},$$
$$(\ ,\ ) = \frac{-2}{(\tanh(\_C1 + \frac{1}{4}\sqrt{\ }\ - \frac{1}{4}\ \ ) - 1)},$$
$$(\ ,\ ) = \frac{2}{(1 + \tanh(-\_C1 + ^{1}}$$

- 1 sech/cosh type solution:

$$(\ , \ , \ ) = \frac{-2}{(\mathrm{sech}(\ _3 + \ ))^2} + \frac{-1}{\mathrm{sech}(\ _3 + \ )} + \ _0 + \ _1 \mathrm{sech}(\ _3 + \ ) + \ _2 \left(\mathrm{sech}(\ _3 + \ )\right)^2.$$

- 1 csc/sin type solution:

$$(\ , \ , \ ) = \frac{-2}{(\csc(\ _3 + \ ))^2} + \frac{-1}{\csc(\ _3 + \ )} + \ _0 + \ _1$$

/F1) 1 Tf0.7797 0 TD(1)TjET217.776 649.23 49.896 0.46796 refBT9.962

3

Example 4. (See [1].) The Zakharov–Kuznetsov KdV-type equation $+ \quad + \quad + \quad + \quad = 0$.

For this $(3+1)$-dimensional PDE, the new package TWS obtains 27 nontrivial solutions. They are as follows.

- 2 rational type solutions:

$$(\; , \; , \; , \;) = -12 \frac{{}^{2}_{2} + {}^{2}_{1} + {}^{2}_{3}}{(\;_{1} + \;_{2} + \;_{3} - \;_{1}\;_{0} + \;)^2} + \;_{0}.$$

$$(\; , \; , \; , \;) = \frac{-2}{2} + \frac{-1}{} + \;_{0} + \;_{1} + \;_{2}(\;)^2, \quad \text{where} \quad = \;_{2} + \;_{3} + \; .$$

- 1 exponential type solution:

$$(\; , \; , \; , \;) = \frac{-2}{(\;^{2} + \;^{3} + \;)^2} + \frac{-1}{\;^{2} + \;^{3} +} + \;_{0} + \;_{1} \;^{2 + 3 +} + \;_{2}(\;^{2 + 3 +})^2.$$

- 4 tan type solutions:

$$(\; , \; , \; , \;) = -12 \frac{{}^{2}_{2} + {}^{2}_{1} + {}^{2}_{3}}{(\tan(\;_{1} + \;_{2} + \;_{3} + (-8 \;^{2}_{3} \;_{1} - 8 \;^{2}_{2} \;_{1} - 8 \;^{3}_{1} - \;_{1}\;_{0}) + \;))^2} + \;_{0}$$

- 4 JacobiCN type solutions:

$$(\; , \; , \; , \;) = \frac{-2}{(\mathrm{JacobiCN}(\; , \omega))^2} + \frac{-1}{\mathrm{JacobiCN}(\; , \omega)} + \;_0 + \;_1 \mathrm{JacobiCN}(\; , \omega) + \;_2 \left(\mathrm{JacobiCN}(\; , \omega)\right)^2, \quad \text{where}$$

$= \;_2 \; T \; \partial \; . \quad T \; \partial(/)/T \quad T \; \partial \; . \quad T \; \partial(\;)Tj \; \partial/T \quad T \; \partial \; . \quad . \quad . \quad . \quad T \; \partial(\;)Tj \quad T \; \partial(\quad T \; \partial \; . \; T \; .T$

- 1 tan type solution:

$$(\ ) = \ _0 + \ _1 \tan(\ ) + \ _2 (\tan(\ ))^2, \qquad = -k(\quad . \ . \ . \quad T \quad \partial(\ ) Tj \ \partial \ . \qquad . \qquad . \qquad /$$

[16] Y.P. Liu, Z.B. Li, Chin. Phys. Lett. 19 (2002) 1228.